The application of wave-activity conservation laws to cloud resolving model simulations of multiscale tropical convection

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What is a wave-activity conservation law?

- A wave-activity conservation law is a relation of the form:

\[
\frac{\partial A}{\partial t} + \nabla \cdot F = 0
\]

\(A\) is a quantity that is conserved in the absence of forcing and dissipation (in contrast to disturbance energy and enstrophy).

- Useful in the study of disturbances \(\xi'\) (waves, eddies) to some background flow \(X\) i.e. \(\xi = X + \xi'\) with \(A \sim O(\xi'^2)\).

- In midlatitudes one takes \(\xi = X(y, z, t) + \xi'(x, y, z, t)\) and defines \(A^{M_x} = \bar{q}'^2/2Q_y\), which satisfies the conservation law:

\[
\frac{\partial A^{M_x}}{\partial t} + \nabla \cdot F = S
\]

with \(\mathbf{F}^{M_x} = -u'v' \mathbf{j} + \frac{f_0}{N^2} v' b' \mathbf{k}; \quad S = \frac{1}{Q_y} S' q'\)

where \(Q\) and \(q'\) are the background and disturbance potential vorticity and \(S\) includes both sources and sinks \(S'\).
Application to large-scale circulations

- Evolution of pseudomomentum flux during a baroclinic life cycle:

\[ \nabla \cdot \mathbf{F}_M < 0 \]

\[ \frac{\partial A_M}{\partial t} < 0 \]

\[ \nabla \cdot \mathbf{F}_M = - \frac{\partial A_M}{\partial t} \]

→ Transience of pseudomomentum density explains flux divergence

Held & Hoskins (1985)
Application to the mesoscale

- On mesoscales one can consider disturbances to a vertically dependent shear-stratified flow.
  \[ \xi = X(z) + \xi'(x, y, z, t). \]
- Disturbance is the mesoscale convection and waves and the background is a shear-stratified flow.

- The 2D pseudomomentum \( A^M_x \) wave-activity conservation law for a \([\bar{u}(z), \bar{\theta}(z)]\) background state is (Scinocca & Shepherd 1992):
  \[
  \frac{\partial A^M_x}{\partial t} + \nabla \cdot F^M_x = S^M_x \quad \text{where}
  \]
  \[
  A^M_x = \frac{\rho_0}{\bar{\theta}_z} \omega' \theta' - \frac{1}{2} \frac{\rho_0^2}{(\bar{\theta}_z)^2} \left( \frac{\bar{u}_z}{\rho_0} \right)_z (\theta')^2; \quad F^M_x = \rho_0 u' w'
  \]
  in the small amplitude limit (can be generalized to finite amplitude).
  \[
  \rightarrow \text{Shaw & Shepherd (2008, 2009) derived the 3D conservation laws and showed how the conservation laws impact the resolved-scale flow in a climate model.} \]
Application to mesoscale momentum transfer

- Pseudomomentum transfers involve two important mesoscale processes:
  - Convective momentum transport (CMT); transport associated with tilted convective structures (Moncrieff 1992).
  - Gravity wave momentum transport (GWMT); transport associated with vertically propagating gravity waves.
  - Processes currently parameterized independently in climate models, which is inconsistent.

- Here we diagnose pseudomomentum wave-activity conservation law from the cloud resolving model simulations of Lane & Moncrieff (2008, 2010).
  - Assess the cause of the vertical momentum flux convergence i.e. transience versus source/sink non-conservative term.
Cloud resolving model simulations

- Cloud resolving model is the anelastic model of Clark et al. (1996)

- Two-dimensional domain: 2000 km wide and 40 km deep with \( \Delta x = 1 \) km and \( \Delta z = 50 \) m to 200 m.

- The imposed thermodynamic sounding was observed over the Tiwi Islands, Australia.

- The imposed zonal wind is either zero (U00) or is westward below 3 km (UMX5B).

- Simulations run for 180 hours
Momentum transfers in unsheared background

- In U00 simulation, shallow convection develops initially and eventually self-organizes into clusters which propagate at 5 m/s without a preferred direction

→ Convection is weakly organized.

Total cloud column

\[ \overline{F_{M_x}(z)} \]

1st EP theorem:

\[ \overline{p'w'} = \rho_0(U - c) \overline{F_{M_x}(z)} \]

GWMT

CMT

→ time

\[ \overline{F_{x}(z)} = \overline{p'w'} \]

\[ \text{dash-dot} = 20-70\text{hr} \]

\[ \text{dash} = 70-120\text{hr} \]

\[ \text{solid} = 120-170\text{hr} \]
Momentum transfers in unsheared background

- Phase speed versus height co-spectra for U00:

\[
\frac{1}{\rho_0} F_{M_x}^*(z)
\]

\[
\frac{1}{\rho_0} A_{M_x}
\]

\[
\frac{1}{\rho_0} F_{M_x}(z)
\]

\[
\frac{1}{\rho_0} A_{M_x}
\]

- Symmetric spectrum except between 120-170hr
- Flux changes sign above convectively active region
- Flux broadens with height
- Pseudomomentum density $A_{M_x}$ changes between 10-15km and below 2km

Lane (2008)
Momentum transfers in unsheared background

- Phase speed versus height co-spectra for U00:

\[
\frac{1}{\rho_0} F^M_x(z) \quad \frac{1}{\rho_0} A^M_x
\]

Colors indicate upward propagation

\(\diamond\) Upward flux does not change sign below convectively active region

\(u'w'\) 120-170hr

Colors indicate upward propagation

\(\diamond\) Upward flux does not change sign below convectively active region
Momentum transfers in unsheared background

- Pseudomomentum flux convergence and its contributions

\[
\frac{1}{\rho_0} \frac{\partial F_{Mx}}{\partial z} \quad \text{(solid)}
\]

\[
\frac{1}{\rho_0} \frac{\partial A M_{x}}{\partial t} \quad \text{(dash-dot)}
\]

\[
F_{Mx}(z) \quad \text{(solid)}
\]

\[
\int S_{Mx} dz \quad \text{(dashed)}
\]

\[
\int \frac{\partial A M_{x}}{\partial t} dz \quad \text{(dash-dot)}
\]

- Time → 20-70hr
- 70-120hr
- 120-170hr

- Source/sink term dominates vertical transfer except between 12-14 km
- Divergence/convergence pattern below 10 km

\[
\frac{\partial F_{Mx}}{\partial z} = S M_{x} - \frac{\partial A M_{x}}{\partial t}
\]

- Source/sink term dominates but there is cancellation
- Transient term usually > 0 and significant up to 20 km
Momentum transfers in unsheared background

- Phase speed versus height co-spectra for U00:

\[
\frac{1}{\rho_0} S^{M_x} = \frac{1}{\rho_0} \frac{\partial F^{M_x}}{\partial z} = S^{M_x} - \frac{\partial A^{M_x}}{\partial t}
\]

- Divergence/convergence pattern consistent with a source between 3-10km and a sink above 10km

- Transience provides a source at all vertical levels

\[
\text{contour} = 0.025 \text{ m/s/day}
\]
Momentum transfers in unsheared background

- Phase speed versus height co-spectra for U00:

\[ \frac{1}{\rho_0} \int \mathcal{S} \mathcal{M}_x \]

\[ \frac{1}{\rho_0} \frac{\partial A \mathcal{M}_x}{\partial t} \]

Time →

20-70hr 70-120hr 120-170hr

○ Divergence/convergence pattern consistent with a source between 3-10km and a sink above 10km

○ The convergence for \( c > 0 \) below 5km is not consistent with upward propagation

\[ \frac{\partial F \mathcal{M}_x}{\partial z} = \mathcal{S} \mathcal{M}_x - \frac{\partial A \mathcal{M}_x}{\partial t} \]

○ Transience provides a source at all vertical levels

○ Transience contribution consistent with upward propagation

contour = 0.025 m/s/day
Momentum transfers in unsheared background

- Pseudomomentum flux convergence involves:
  - CMT which peaks at ±2 m/s below 8 km.
  - GWMT which peaks at ±4-6 m/s from the surface up to 20 km.
- Upward propagating part of spectrum is consistent between the two regions.
  - GWMT source is below 7 km.
- Transient signal near convective outflow from 10-14 km which peaks at -4 m/s.
- Source/sink dominates below 8 km.
- Transient signal dominates aloft.

![Contribution to spectra](image)

- Transient contribution peaks at ±5 m/s
- Source/sink contribution peaks between 2-6 m/s
Momentum transfers in the presence of low-level shear

- In the presence of low level wind shear the convection becomes organized; transitions from upshear propagating (positive tilt) to quasi-stationary (upright tilt) and finally to cloud clusters (negative tilt).
Momentum transfers in the presence of low-level shear

- Phase speed versus height co-spectra for UMX5B:

- Spectrum no longer symmetric

- Non-upward flux contribution is zero above 7km

- Upward flux contribution is large below 10km
Momentum transfers in the presence of low-level shear

- Pseudoenergy flux co-spectra for U00 & UMX5B:

Time → 20-70hr 70-120hr 120-170hr

- Spectrum symmetric for U00
- Flux weakens with time
- Broadening of westward part of the spectrum
- Enhanced upward propagation between 0-5 km
- Peak between -5 to -15 m/s
- Slight enhancement of eastward part
Momentum transfers in the presence of low-level shear

- Pseudomomentum flux convergence and its contributions

\[ \frac{1}{\rho_0} \frac{\partial F_{Mx}^z}{\partial z} \] (solid)
\[ \frac{1}{\rho_0} \frac{\partial A_{Mx}^z}{\partial t} \] (dash-dot)
\[ \frac{\partial F_{Mx}^z}{\partial z} = S_{Mx} - \frac{\partial A_{Mx}^z}{\partial t} \]

\[ \int S_{Mx}^z dz \] (dashed)
\[ \int \frac{\partial A_{Mx}^z}{\partial t} dz \] (dash-dot)

\[ 1 \rho_0 \frac{\partial F_{Mx}^z}{\partial z} \]
\[ \frac{1}{\rho_0} \frac{\partial A_{Mx}^z}{\partial t} \]

\[ F_{Mx}^z \] (solid)
\[ A_{Mx}^z \]

Time → 20-70hr  70-120hr  120-170hr

- Transfers are much stronger in the troposphere
- Large transfers near the surface
- Additional vorticity term
- Balance between \( S_{Mx} \) and \( A_{Mx}^z \) transience
- Flux \( F_{Mx}^z \) is on the order of \( A_{Mx}^z \) transience
Momentum transfers in the presence of low-level shear

- Phase speed versus height co-spectra for UMX5B:

\[
\frac{1}{\rho_0} S M_x
\]

\[
- \frac{1}{\rho_0} \frac{\partial A M_x}{\partial t}
\]

\[
\frac{\partial F}{\partial z} = S M_x - \frac{\partial A M_x}{\partial t}
\]

- Divergence/convergence pattern consistent with a source between 3-10km and a sink above 10km
- The convergence for \( c > 0 \) below 5km is not consistent with upward propagation
- Transience provides a source at all vertical levels
- Transience contribution consistent with upward propagation

contour = 0.025 m/s/day
Momentum transfers in the presence of low-level shear

- Phase speed versus height co-spectra for UMX5B:

\[
\frac{1}{\rho_0} S M_x \quad \frac{-1}{\rho_0} \frac{\partial A M_x}{\partial t}
\]

\[
\begin{aligned}
\text{Time} & \rightarrow 20-70hr & 70-120hr & 120-170hr \\
\end{aligned}
\]

- Divergence/convergence pattern consistent with a source between 3-10km and a sink above 10km
- The convergence for \( c > 0 \) below 5km is not consistent with upward propagation
- Transience provides a source at all vertical levels
- Transience contribution consistent with upward propagation

\[
\frac{\partial F M_x}{\partial z} = S M_x - \frac{\partial A M_x}{\partial t}
\]

contour = 0.025 m/s/day
Momentum transfers in the presence of low-level shear

- CMT transitions from positive to negative.
  → Consistent with convective regimes (Houze 2004).
- GWMT shows similar transition (LM10).
- Both show a broadened westward spectrum between 10-20 m/s.
  → Transience is responsible for the broadened spectrum
  → Associated with transient updrafts in the organized convection.

20-70hr: CMT > 0 GMT > 0
120-170hr: CMT < 0 GMT < 0

- Transient contribution has a much broader westward spectrum
- Source/sink contribution has a weaker broadening
Momentum transfers in the presence of low-level shear

- When the convection is organized, transient updrafts move rearward in the cloud frame creating a transient signal.

\[ F_{Mx}(z) < 0 \]

- Spectral bias arises because tilted source projects onto waves propagating in the direction of tilt.

- Transient updrafts move rearward relative to the core.

→ Effect of transient updrafts noted by Fovell et al. (1992)
Conclusions

• Pseudomomentum wave-activity conservation law can be used to understand the mesoscale transfers of momentum associated with convection and gravity waves.
  → CMT and GWMT contributions can be isolated using upward propagation criteria (first Eliassen-Palm theorem).
  → Can be used to isolate CMT and GWMT and their sources and sinks in cloud resolving model simulations.

• Low-level shear significantly impacts the CMT and GWMT spectrum both above and below the tropopause.
  → Leads to a broadened phase speed spectrum.
  → Transience from the organized convection can be connected to signals above the tropopause and cannot be ignored.

• CMT and GWMT should not be treated independently in climate models.
  → GWD parameterizations should account include transient sources and the effects in the source region.
Application to mesoscale momentum transfer

- Dispersion relation

\[(\sigma - Uk)^2 - \frac{kU_{zz}}{k^2 + m^2}(\sigma - Uk) - \frac{N^2k^2}{k^2 + m^2} = 0\]

- Long wave limit, e.g. \(k^2 + m^2 \ll U_{zz}/N^2\), vorticity waves:

\[\sigma \sim Uk + \frac{kU_{zz}}{k^2 + m^2}\]

- Short wave limit, e.g. \(k^2 + m^2 \gg U_{zz}/N^2\), gravity waves:

\[\sigma \sim Uk + \frac{Nk}{(k^2 + m^2)^2}\]
Application to mesoscale momentum transfer

- Pseudoenergy

\[ A^\varepsilon = |\mathbf{v}'|^2 + \frac{\rho_0}{2} \left[ \frac{1}{(\bar{\theta}_z)^2} - \frac{\rho_0 \bar{u}}{(\bar{\theta}_z)^2} \left( \frac{U_z}{\rho_0} \right)_z \right] (\theta'^2)_z^2 + \frac{\rho_0 \bar{u}}{\theta_z} \omega \theta' \]

with

\[ F^\varepsilon_{(z)} = c_p \rho_0 \pi' \mathbf{w}' + \rho_0 u' \mathbf{w}' \]
The 2D pseudomomentum $A^{M_x}$ wave-activity conservation law for a $[\bar{u}(z), \bar{\theta}(z)]$ background state is:

$$\frac{\partial A^{M_x}}{\partial t} + \nabla \cdot F^{M_x} = 0$$

where

$$A^{M_x} = \frac{\rho_0}{\theta_z} \omega' \theta' + (\bar{w} + \omega') \int_0^{\theta'} \left[ R(\bar{\theta} + \eta) - R(\bar{\theta}) \right] \, d\eta$$

$$- \rho_0 \int_0^{\theta'} \left[ S(\bar{\theta} + \eta) - S(\bar{\theta}) \right] \, d\eta$$

$$F_z^{M_x} = \rho_0 u' w' + w' A^{M_x}$$
Application to mesoscale momentum transfer

- Phase speed versus height co-spectra for UMX5B:

\[ F_{M_x}(z) \]

\[ F_{E(z)} - \bar{u}F_{M_x}(z) \]

- Spectrum is symmetric
- Non-upward flux contribution is zero above 7km
- Upward flux contribution is large below 10km
• In the presence of mid-level wind shear the convection becomes organized very quickly; cloud clusters (negative tilt) emerge and propagate eastward.
Application to mesoscale momentum transfer

- Contribution of non-linear flux term to domain-averaged profiles

Time $\rightarrow$ 20-70hr, 70-120hr, 120-170hr

\[
\frac{\partial F_{Mx}^{(z)}}{\partial z}
\]
Sources and sinks

- The source/sink terms in the wave-activity conservation laws involve the source/sink terms on the mesoscale

\[
\bar{S}^E = \mathbf{v}' \cdot \mathbf{S_v} + \rho_0 \left[ \frac{1}{(\theta_0)^2 N^2} - \frac{\rho_0 \mathbf{u}}{((\theta_0) z)^2} \cdot \left( \frac{\mathbf{u}_z}{\rho_0} \right)_z \right] \theta' \bar{S}_\theta - \frac{\rho_0}{\Theta_z} \mathbf{u} \cdot \left[ \omega'^\perp \bar{S}_\theta - \theta' (\nabla \times \mathbf{S_v})^\perp \right]
\]

\[
\bar{D}^{P_{x,y}} = -\frac{(\rho_0)^2}{((\theta_0) z)^2} \left( \frac{\mathbf{u}}{\rho_0} \right)_z \theta' \bar{S}_\theta - \frac{\rho_0}{\Theta_z} \left[ \omega'^\perp \bar{S}_\theta - \theta' (\nabla \times \mathbf{S_v})^\perp \right].
\]

with

\[
\bar{S}^E = \mathbf{v}' \cdot \mathbf{S_v} + \rho_0 \theta' \bar{S}_\theta/(\theta_0)^2 N^2 - \mathbf{u} \cdot \bar{D}^{P_{x,y}}
\]

- The second term in \( \bar{S}^E \) is recognized as a term in the available potential energy budget since \( \rho_0 \theta' \bar{S}_\theta/(\theta_0)^2 N^2 = \rho_0 \theta' \mathbf{w}'/\theta_0 \).