Non-stationery trapped gravity lee waves, and their impact on the large-scale

Matt Hills and Dale Durran
Department of Atmospheric Science, University of Washington
Introduction

Possible ways for trapped lee waves to decay:
- Dissipation
- Leakage
- Untrapping (i.e. become vertically propagating)

Impact on large scale depends on type of decay

Goals:
- How trapped waves evolve in a time-varying flow.
- How the large-scale flow is modified by the waves.
Model setup

- 1200km x 1200km horizontal domain.
- 20km vertical depth.
- Translating square-wave slowly crosses the mountain
  - $\tau = 33.3$ hours.
- Stability interface at 3km
  - $N_L = 0.0118 \text{s}^{-1}$,
  - $N_U = 0.003 \text{s}^{-1}$.
Vertical velocity animation

Heigh (km)

T = 10.17 hrs
T = 10.33 hrs
T = 10.5 hrs
T = 10.67 hrs
T = 10.83 hrs
T = 11 hrs
T = 11.17 hrs
T = 11.33 hrs
T = 11.5 hrs
T = 11.67 hrs
T = 11.83 hrs
T = 12 hrs
T = 12.17 hrs
T = 12.33 hrs
T = 12.5 hrs
T = 12.67 hrs
T = 12.83 hrs
T = 13 hrs
T = 13.17 hrs
T = 13.33 hrs
T = 13.5 hrs
T = 13.67 hrs
T = 13.83 hrs
T = 14 hrs
T = 14.17 hrs
T = 14.33 hrs
T = 14.5 hrs
T = 14.67 hrs
T = 14.83 hrs
T = 15 hrs
T = 15.17 hrs
T = 15.33 hrs
T = 15.5 hrs
T = 15.67 hrs
T = 15.83 hrs
T = 16 hrs
T = 16.17 hrs
T = 16.33 hrs
T = 16.5 hrs
T = 16.67 hrs
T = 16.83 hrs
T = 17 hrs
T = 17.17 hrs
T = 17.33 hrs
T = 17.5 hrs
T = 17.67 hrs
T = 17.83 hrs
T = 18 hrs
T = 18.17 hrs
T = 18.33 hrs
T = 18.5 hrs
T = 18.67 hrs
T = 18.83 hrs
T = 19 hrs
T = 19.17 hrs
T = 19.33 hrs
T = 19.5 hrs
T = 19.67 hrs
T = 19.83 hrs
T = 20 hrs
T = 20.17 hrs
T = 20.33 hrs
T = 20.5 hrs
T = 20.67 hrs
T = 20.83 hrs
T = 21 hrs
T = 21.17 hrs
T = 21.33 hrs
T = 21.5 hrs
T = 21.67 hrs
T = 21.83 hrs
T = 22 hrs
T = 22.17 hrs
T = 22.33 hrs
T = 22.5 hrs
T = 22.67 hrs
T = 22.83 hrs
T = 23 hrs
T = 23.17 hrs
T = 23.33 hrs
T = 23.5 hrs
T = 23.67 hrs
T = 23.83 hrs
T = 24 hrs
T = 24.17 hrs
T = 24.33 hrs
T = 24.5 hrs
T = 24.67 hrs
T = 24.83 hrs
T = 25 hrs
T = 25.17 hrs
T = 25.33 hrs
T = 25.5 hrs
T = 25.67 hrs
T = 25.83 hrs
T = 26 hrs
T = 26.17 hrs
T = 26.33 hrs
T = 26.5 hrs
T = 26.67 hrs
T = 26.83 hrs
T = 27 hrs
T = 27.17 hrs
T = 27.33 hrs
T = 27.5 hrs
T = 27.67 hrs
T = 27.83 hrs
T = 28 hrs
T = 28.17 hrs
T = 28.33 hrs
T = 28.5 hrs
T = 28.67 hrs
T = 28.83 hrs
T = 29 hrs
T = 29.17 hrs
T = 29.33 hrs
T = 29.5 hrs
T = 29.67 hrs
T = 29.83 hrs
T = 30 hrs
T = 30.17 hrs
T = 30.33 hrs
T = 30.5 hrs
T = 30.67 hrs
T = 30.83 hrs
T = 31 hrs
T = 31.17 hrs
T = 31.33 hrs
T = 31.5 hrs
T = 31.67 hrs
T = 31.83 hrs
T = 32.17 hrs
T = 32.33 hrs
T = 32.5 hrs
T = 32.67 hrs
T = 32.83 hrs
T = 33 hrs
T = 33.17 hrs
T = 33.33 hrs
T = 33.5 hrs

w (ms^-1)
Ray tracing

- Wave packets have been followed through the simulation using ray tracing.
  - Packet motion assumed 2D(x,t).

- Dispersion relation: \( \cot(S_L d) = -\frac{S_U}{S_L} \), where \( S_U = \left( k^2 - \frac{N_U^2}{(c-\bar{u})^2} \right)^{\frac{1}{2}} \) and \( S_L = \left( \frac{N_L^2}{(c-\bar{u})^2} - k^2 \right)^{\frac{1}{2}} \)

- Group velocity: \( c_{g_x} = \frac{\bar{u}}{} + \frac{S_L^4 S_U d (c-\bar{u})^3}{N_L^2 S_L^2 S_U d + k^2 (N_L^2 - N_U^2) \sin^2(S_L d)} \)

- Parameters of interest: wavelength, position, and intrinsic frequency (\( \tilde{\omega} \)).
  - Waves are trapped provided: \( N_U < |\tilde{\omega}| < N_L \), where \( \tilde{\omega} = (c-\bar{u})k \).
Ray tracing

- Decaying waves are untrapped by the background flow slowly varying with time.

  ▪ Untrapping can be described by: \( \frac{D \tilde{\omega}}{Dt} = k \frac{D \tilde{u}}{Dt} \), where \( \frac{D}{Dt} = \frac{d}{dt} + \tilde{u} \frac{d}{dx} \)

  ▪ Before peak flow at mountain top, this says \( \frac{D \tilde{\omega}}{Dt} < 0 \), and \( \tilde{\omega} \) remains bounded throughout the simulation.

  ▪ After peak flow at mountain top, this says \( \frac{D \tilde{\omega}}{Dt} > 0 \), meaning \( \tilde{\omega} \) eventually leaves the bounds, and waves become untrapped.

- Behavior is seen to be highly dependent on the structure of the background flow.
Ray tracing
Illustration of wave untrapping
Illustration of wave untrapping

\[ N_L = 1.18 \times 10^{-2} \text{ s}^{-1}, \quad N_U = 3.0 \times 10^{-3} \text{ s}^{-1} \]

\[ z = 3 \text{ km} \]

\[ |\tilde{\omega}| = 2.8 \times 10^{-3} \text{ s}^{-1}, \quad |\tilde{\omega}| = 3.3 \times 10^{-3} \text{ s}^{-1}, \quad |\tilde{\omega}| = 3.9 \times 10^{-3} \text{ s}^{-1} \]
Vertical momentum flux

- Vertical gradient of momentum flux can tell us about changes to the large-scale flow.
  \[ \frac{d\langle \rho_0 \bar{u} \rangle}{dt} = - \frac{d\langle \rho_0 u'w' \rangle}{dz} \]

- If the flux becomes more negative with height:
  - Background flow is accelerated.

- If the flux becomes less negative with height:
  - Background flow is decelerated.
Vertical momentum flux

Horizontally integrated momentum flux $\langle \rho \theta \omega \rangle$

Height (km)

Time (hrs)

Waves untrap

Legend:

- 0
- $-1 \times 10^9$
- $-2 \times 10^9$
- $-3 \times 10^9$
- $-4 \times 10^9$
- $-5 \times 10^9$
- $-6 \times 10^9$
- $-7 \times 10^9$
Pressure drag

- Trapped wave train requires a drag on the flow (Broad 2002).
  - Pressure drag balances the vertical momentum flux forced by the waves seen on the previous slide.

- Features of interest:
  - Note that the drag profile is non-symmetric about the time of peak wind.
  - Dominant component of the total drag comes from the mesoscale part of the flow.

Considering flux magnitude from previous slide

- ~5/7 of mesoscale drag comes from vertically propagating waves.
- ~2/7 comes from the trapped waves.
Perturbations to the large-scale

- Background zonal flow is modified at all levels by trapped wave train.
  - Changes are both local and far reaching.

- $u$ changed by $\approx 10\%$ in some parts of the domain.
  - Perturbations away from the waves may be $\approx 0.5\text{ms}^{-1}$, forced by our (km) 250m mountain.
Trapped waves can become untrapped through changes in the background wind field.

- Waves forced when the background flow at the mountain is accelerating remain trapped.
- Waves forced when the flow at the mountain is decelerating become untrapped, and decay.

Changes in the mean flow occur due to the trapped waves.

- Flow is decelerated while the trapped wave train is present.
- Background flow is accelerated following wave untrapping.

Non-negligible changes are made to the background wind field.
Perturbations to the large-scale

- Significant changes are seen at all levels.
  - Most notable modification is observed in slower regions of the background flow, at the edges of the localized jet.

- Broad pattern away of the wave train:
  - Flow to the south of the ridge is accelerated.
  - Flow to the north of the ridge is decelerated.
Stratosphere run
Different mountain heights
Satellite imagery

1715 UTC

1830 UTC

2045 UTC

2230 UTC