Gravity waves emitted from jets: lessons from idealized simulations

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From observations, it has been known that jet exit regions often exhibit intense inertia-gravity wave activity.

Simulations of idealized baroclinic life cycles have confirmed this and shown that specific processes for propagation (i.e. 'wave capture') play an important role.

A simpler flow exhibits similar wave generation: dipoles.

The generation of inertia-gravity waves in the front of a dipole has been explained as the linear response, on the background of a balanced dipole, to the small discrepancies between the balanced and the full tendencies for wind and potential temperature.
1. Dipole simulations

Why a dipole?

- **Simpler flow:** stationary in the co-moving frame
- **Complex enough:** model of jet-streaks (includes a 'jet exit')

**Numerical Setup**

Initial condition = exact Surface Quasi-Geostrophic (SQG) dipole

Model = Boussinesq, Primitive Equations,

\( x, y \) periodic domain (3000 x 3000 km), 15 km deep, sponge layer

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Snyder et al 2007
Emission mechanism

Waves are small perturbations to a nearly balanced flow:

1. Separate flow:

\[ u = \overline{u} + u' \]

2. Linearize about balanced dipole

3. Evolve perturbations:
   
   A without forcing (homogeneous solutions)
   
   B with forcing (forced solutions)

Snyder et al 2009
Separate the flow:

\[
\partial_t \overline{u} + \partial_t u' + \overline{u} \cdot \nabla \overline{u} + \overline{u} \cdot \nabla u' \\
+ u' \cdot \nabla \overline{u} + u' \cdot \nabla u' - f \overline{v} - f v' + \partial_x \overline{\phi} + \partial_x \phi' = 0 ,
\]

Rewrite as a linear operator* forced by terms from the balanced flow:

\[
\partial_t u' + \overline{u} \cdot \nabla u' + u' \cdot \nabla \overline{u} - f v' + \partial_x \phi' = \mathcal{F}_u + O(u'^2) ,
\]

where** \[
\mathcal{F}_u = - \left( \partial_t \overline{u} + \overline{u} \cdot \nabla \overline{u} - f \overline{v} + \partial_x \overline{\phi} \right) .
\]

* it is crucial to note that the linear operator is not the operator for waves on a fluid at rest, it has non-constant coefficients, given by the background dipole flow

** note that the forcing is large-scale:

\[
\begin{align*}
F_u & \quad F_v & \quad F_\theta
\end{align*}
\]
A Homogeneous solutions (= ageostrophic instability)?

\[ \partial_t u' + \mathbf{u} \cdot \nabla u' + u' \cdot \nabla \mathbf{u} - f \mathbf{v}' + \partial_x \phi' = 0 \]

No growth, **but** signal with the relevant structure comes out

- not an instability,
- basic flow is crucial
Forced solutions?

\[ \partial_t u' + \mathbf{u} \cdot \nabla u' + u' \cdot \nabla \mathbf{u} - f v' + \partial_x \phi' = \mathcal{F}_u \]

**Forced linear prediction:**

**PE – QG:**

Snyder et al 2009
Wave quantification

Relative to previous studies (Snyder et al 07, 09), we use here the QG+1 approximation followed by spatial filtering to better identify the gravity waves in all fields:

Example of decomposition of $w$

Decomposition of $u$ (left) and $v$ (right) for dipoles of different strengths (5, 10 and 15m/s)
Scaling the waves relative to the dipole strength

Dipole strengths: $U = 2.5, 5, 7.5, 10, 12.5$ and $15\text{ m/s}$

Ways to quantify the waves: maxima of $u'$, $v'$, $w'$, and the square root of perturbation $KE$

Behavior previously found for $w'$ not recovered (Snyder et al 07)

Rather robust behavior found for $u'$, $v'$ and the square root of perturbation $KE$:

- at low $Ro$, signal corresponds to order 2 balanced corrections (hence a slope of $\sim 2$)
- break around $U=7.5 \text{ m/s}$
- at moderate $Ro$, signal corresponds to GW, with a slope of $\sim 3.5$

*Figure 3: Blue line: maximum of $u'$, black line: maximum of $v'$, red line: maximum of $w'$, and green line $\sqrt{KE}$. Filtering used: Hann window with $n = 12$. All curves are normalized by value for $U_0 = 7.5 \text{ m s}^{-1}$. Log log plot.*
Diagnostics used to identify emission from jets and fronts

Lagrangian Rossby number:

$$R_{OL} = \frac{|dV/dt|}{f|V|} = \frac{|V_{ag}|}{|V|}$$

Cross-stream Lagrangian Rossby number (Koch and Dorian, 1988, Zhang et al 2000):

$$R_{OL}^\perp \approx \frac{|V_{ag}^\perp|}{|V|}$$

Frontogenesis Function

$$F \equiv \frac{1}{2} \frac{D|\nabla \Theta|^2}{Dt}$$

Residual of Nonlinear-Balance Equation (Zhang et al 2000, Plougonven and Zhang 2007)

$$\Delta NBE = 2J(u,v) + f\zeta - \alpha \nabla^2 p$$

Okubo-Weiss parameter, as an indication of wave-capture (Bühler and McIntyre 2005, Plougonven and Snyder 2007, Wang and Zhang 2010)

$$\sigma_n = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$$

$$\sigma_s = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\sigma = \sqrt{\sigma_n^2 + \sigma_s^2}$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$W = \sigma^2 - \zeta^2$$
Relevance of the Okubo-Weiss parameter relative to the wave-packets appearing in:

Dipole simulations

Baroclinic life cycles

Colors show $w$ (left) or the divergence of horizontal wind (right), while black contours indicate positive maxima of the Okubo-Weiss parameter.
N.B. The above diagnostics are either linear (Rossby numbers, OW parameter) or quadratic.

Hence, each diagnostic keeps the same structure for dipoles of different strengths, with an amplitude given by the Rossby number.

The diagnostics are shown here only for the dipole with $U=10\text{m/s}$.
2. Baroclinic life cycles

Standard (cyclonic) development

Anticyclonic shear

Strong anticyclonic shear

Reference simulation described in Plougonven & Snyder 2005, 07
Diagnostics calculated in the reference simulation
Relation of the inertia-gravity waves and the Okubo-Weiss parameter

Life cycle with strong anticyclonic shear shows that the results obtained in the reference run and for the dipole do not always apply in the same way:
Summary

Jet Exit Region EMIssion (JEREMI) occurs systematically in both baroclinic life cycles and dipole simulations.

JEREMI is explained in the dipole as a linear response, on the background of a dipole flow, to the residual tendencies (the residual when a balanced dipole solution is injected in the full equations).

Dipole simulations with dipoles of increasing strength have allowed to quantify the gravity waves as a function of Rossby number (or dipole strength).

QG+1 has been used to isolate the balanced part of the flow, and hence identify more precisely the GWs.

Different diagnostics that have been put forward in previous studies have been tested for:
- the relevance of their location relative to the excited waves;
- a relationship between them and the amplitude of the GWs.

In the dipole simulations, these diagnostics fail to determine the appearance of the GWs at a certain Rossby number. The location of the Okubo-Weiss parameter often predicts where the main core of the Gws will be. The location of the Lagrangian Rossby numbers or other traditional diagnostics do not appear easy to relate to the GWs.

References