# Momentum Fluxes of Gravity Waves Generated by Variable Froude Number Flow over Idealized and Realistic Three-Dimensional Orography


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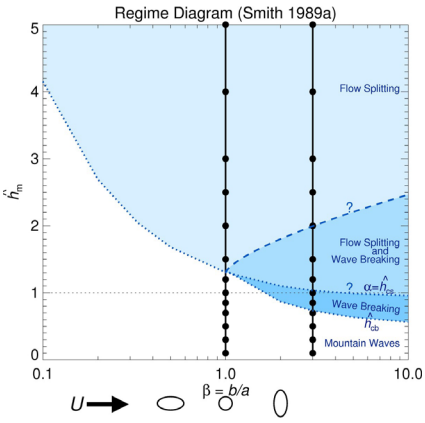



**Elliptical Three-Dimensional Hill**

$$h(x,y) = \frac{h_m}{[1+(x/a)^2+(y/b)^2]^{3/2}}.$$

**Constant Upstream Flow Profile**

$U = 10$ m s$^{-1}$ $\quad N = 0.01$ rad s$^{-1}$

**Normalized Hill Height** (inverse Froude number)

$$\hat{h}_m = Nh_m/U = \mathrm{Fr}^{-1}$$

**Obstacle Aspect Ratio** $\beta = \frac{b}{a}$

**Numerically Modeled Cases**

## Primary Modeling & Diagnostic Tools

### (1) Advanced Research Weather Research & Forecasting (WRF) Model V3.0

- 900km x 900 km x 30km domain, Δx = Δy = 1.5 km, Δz ≈ 125m
- Sponge layer z=15-30 km (ramped implicit Rayleigh friction: Klemp et al. 2008)
- Free-slip lower boundary, $f$ = 0, 1.5-order turbulent closure scheme

### (2) Diagnostic Fourier Transform Method (Fourier-ray solutions)

WRF solution at $z=z_i$ and $t$=24 hours ($t\to\infty$ limit)

$$w(x,y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{w}(k,l,z)\, e^{i(kx+ly)}\, dk\, dl$$

$$\tilde{w}(k,l,z) = \left[\frac{\rho(z_i)}{\rho(z)}\right]^{1/2} \tilde{w}(k,l,z_i)\, e^{im(z-z_i)}$$

$$m = -(k^2+l^2)^{1/2}\left(\frac{N^2}{\hat{\omega}^2}-1\right)^{1/2}$$

### Spatial Gravity-Wave Momentum Flux Solutions

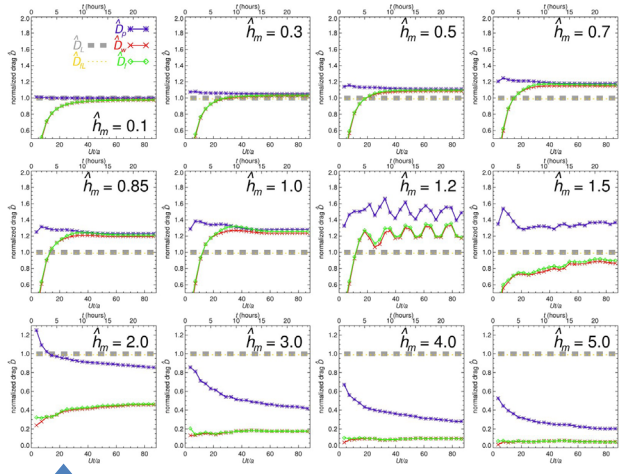
$$\overline{u'w'}(x,y,z) = \frac{1}{4}(\hat{u}\hat{w}^* + \hat{u}^*\hat{w});$$

$$\overline{v'w'}(x,y,z) = \frac{1}{4}(\hat{v}\hat{w}^* + \hat{v}^*\hat{w}),$$

$\hat{u}$, $\hat{v}$, and $\hat{w}$ are complex spatial realizations of the Fourier-ray solutions above for all three velocity component. Complex forms follow from a simple Hilbert Transform operation [see Eckermann et al. 2010 for further details].

## Time Series of Area-Integrated Momentum Flux

### Axisymmetric Obstacle: β = 1 (Drag $D$ Normalized by Linear Value $D_L$)




**Surface Pressure Drag**

$$\mathbf{D}_p = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p'\left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}\right) dx\, dy.$$

$$\hat{D}_p = \frac{D_p}{D_L}$$

**Linear Drag Solution**

$$|\mathbf{D}_L| = B(\beta)a\rho_0\frac{U^3}{N}\hat{h}_m^2$$

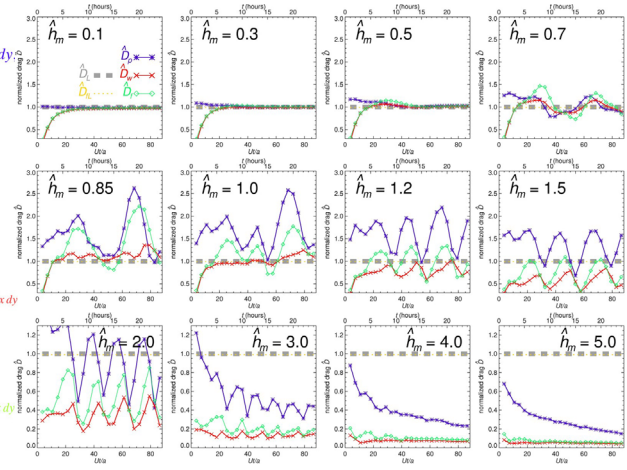
**Drag at z = 6km**

$$\mathbf{D}_w(z) = \rho(z)\int_{-L/3}^{+2L/3}\int_{-L/2}^{+L/2}(u'w', v'w')\, dx\, dy$$

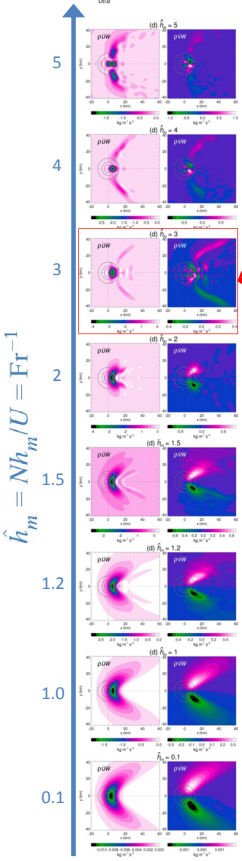
**Fourier-Ray Drag at z = 6km**

$$\mathbf{D}_{\hat{f}}(z) = \rho(z)\int_{-L/3}^{+2L/3}\int_{-L/2}^{+L/2}(\overline{u'w'}, \overline{vw})\, dx\, dy$$

### Elliptical Obstacle: β = 3

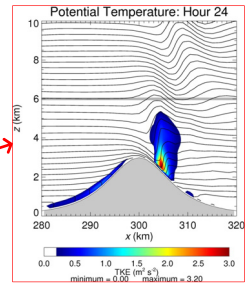



## Momentum Fluxes at $z$ = 6km

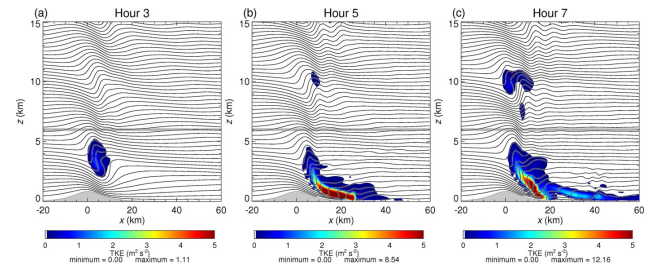



**Wave Generation at Cross-Stream Extremities of Hill Due to Flow Splitting**

**Wave Forcing Dominated by Leeside Collapse of Isentropes**




## Periodic/Vacillating Momentum Flux Responses

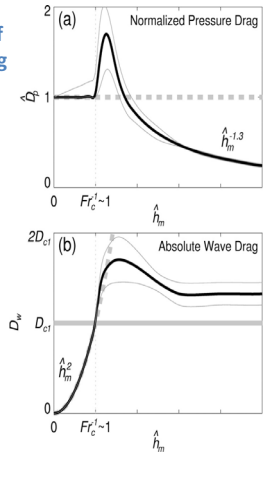


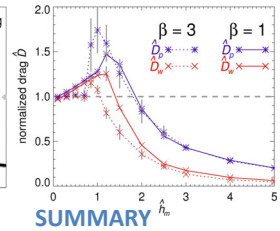

## Bulk Drag Characteristics

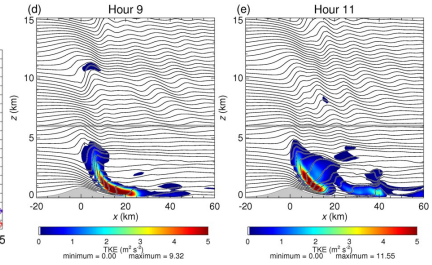
**Contraction of Diverging Wing Pattern**

**Flux Peaks Migrate to Lee Downslope**

**Linear "Ship Waves"**




## SUMMARY

- Surface dynamics transition from linear to high-drag states at $Fr^{-1}$ values predicted to within ~10% by linear transform relations
- Surface drag and gravity-wave momentum flux responses do not equilibrate but vacillate in the high-drag-state range of $Fr^{-1}$ ~ 1-2.5
- Gravity-wave generation at $Fr^{-1} \geq$ 1-1.5 is dominated by leeside collapse of isentropes and has little correlation with the obstacle shape
- Area-averaged gravity-wave momentum flux at $Fr^{-1} \geq 2$ asymptotes to constant values, despite large variations in its spatial morphology
- This constant value at $Fr^{-1} \geq 2$ is quantified to within 10-40% by a simplified analytical formula using a Sheppard dividing streamline estimate
- Coupled Fourier-WRF methodology works and has recently been extended to analysis of waves generated from Hawaii (Lindeman et al. 2010)

## REFERENCES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Smith, R. B. (1989), Mountain-induced stagnation points in hydrostatic flow. Tellus, 41A, 270–274.