Application of the GWR Method to the Tropical Indian Ocean*

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ABSTRACT

The gravity wave retardation (GWR) method is a simple technique that allows layer models to include bottom topography. Here the method is applied, and its accuracy is evaluated, for monthly climatological wind forcing in an Indian Ocean model with realistic bottom topography. This is an extension of previous studies where the GWR method was applied to idealized wind forcing in oceans with idealized basin geometry. Comparison to a model integration with a flat bottom demonstrates that GWR integrations with speedup factors of up to 16 indeed capture the influence of the bottom relief and have less error in the deep volume transports. For a speedup factor of that magnitude, a GWR integration is also found to have less error than a reduced gravity model simulation. It is concluded that integrations using the GWR method give remarkably good results for the upper-layer circulation as well as the deep flow with a speedup factor of up to 8.

1. Introduction

In the open ocean, external, or barotropic, gravity waves propagate with phase speeds exceeding 200 m s⁻¹. For computationally efficient ocean models this implies that this mode should either be entirely removed, or solved by a different method than used for the internal, or baroclinic, modes.

By applying a rigid lid, (e.g., Bryan 1969; Semtner 1986) the external gravity waves are removed, while planetary waves are retained. This has been the classical approach used in ocean modeling. A second method is to apply a mode-splitting technique to ocean models, integrating the barotropic mode with a smaller time step (Simons 1974; Madala and Piacsek 1977). This method has been successfully applied to sigma-coordinate models (Blumberg and Mellor 1987), z-coordinate models (Killworth et al. 1991), and of particular relevance for this study, to the Miami Isopycnic Coordinate Ocean Model (MICOM; Bleck and Smith 1990) and the Hallberg Isopycnal Model (HIM; Hallberg 1997). However, as shown by Higdon and Bennett (1996), the mode splitting as applied in the original MICOM model can lead to instabilities, and a complete mode separation is not

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trivial to obtain. Details about mode splitting in MICOM and HIM can be found in Higdon and Bennett (1996), Higdon and de Szoeke (1997), Hallberg (1997), and Higdon (1999).

In the tropical oceans, where the barotropic modes are of less importance, reduced gravity models may be applied. That method assumes one or more active layers over an infinitely deep abyss and removes all barotropic modes, (e.g., Busalacchi and O'Brien 1980; Jensen 1991).

A simple way to include finite depth in layer models is to reduce the speed of the barotropic gravity waves (Jensen 1996; Tobis 1996). This method of gravity wave retardation (GWR) allows a relatively long time step compared to the undistorted case, while keeping the integration scheme explicit. In Jensen (1996) it was demonstrated that the GWR method gave very good results under steady wind forcing for a flat bottom ocean as well as with bottom topography. Tobis (1996) investigated cases with seasonal wind forcing for two layer models with a flat bottom. However, both studies were limited to idealized ocean basins and forcing.

For steady-state calculations in nonstratified coastal models, Hearn and Hunter (1987) and Hunter (1990a,b) used a reduced value of the acceleration of gravity in order to reach equilibrium solutions with much reduced computational effort. For the stratified ocean, Bryan (1984) devised a method for accelerated convergence toward a steady state by compressing timescales for all ocean waves. However, both these methods distort the solutions severely for time-dependent problems.

The main advantage of the GWR method is its simple implementation. However, since the method does re-

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quire a substantially shorter time step than required for the baroclinic mode, its efficiency compared to modesplitting methods decreases as the number of layers increases. For instance, in MICOM, the computational time for a barotropic mode time step is about the same as for the baroclinic mode for a single layer (Bleck and Smith 1990). If we assume a barotropic gravity wave speed of $c_0 = 240 \text{ m s}^{-1}$ and a sum of internal gravity wave speed and advection velocity of $c_1 = 4 \text{ m s}^{-1}$, we would need $c_0/c_1 = 60$ barotropic time steps per baroclinic time step. However, since MICOM uses a leapfrog scheme for the internal mode and a forward-backward scheme for the barotropic mode, only 30 barotropic steps per baroclinic step is needed. The computational work is then proportional to $N + \frac{1}{2}c_0/c_1$, where N is the number of layers. With the GWR method, the work compared to a single layer is $N\sqrt{\gamma}c_0/c_1$, where $1/\sqrt{\gamma}$ is the speedup factor. For a speedup of a factor of 8, and c_0 and c_1 as above, we find that the GWR method is faster than mode splitting for up to four layers. However if the same numerical scheme is used for both the barotropic and baroclinic modes, the GWR method is most efficient for up to nine layers in this example. Clearly, mode splitting is needed for a large number of vertical layers, and although the GWR method could be applied to the barotropic mode in order to increase the short time step, the benefit of doing so decreases with vertical resolution.

In this paper, the GWR method is applied to the tropical Indian Ocean with monthly climatological wind forcing to determine the errors associated with the method for more realistic circulation models. A comparison with a reduced gravity model is also included to provide a practical lower limit on the acceptable quality of the GWR solutions. One of the main questions is if, by adding bottom topography and using a small GWR factor, we can improve upon the reduced gravity model results, without a substantial increase in computational requirement, or if the upper-layer results might get adversely affected.

The Indian Ocean has a very large seasonal cycle due to the Indian monsoon and the associated ocean currents have high variability, including strong, western boundary currents and seasonal eddy activity. This makes it a challenging test case for applying the GWR method to tropical oceans.

2. The gravity wave retardation method

a. Background

The method of gravity wave retardation as applied here is given in detail by Jensen (1996) and is briefly outlined below. The idea of GWR is based on the method of artificial compressibility (Chorin 1967), in which an artificially reduced value of the sound speed, c_s , is used to slow down the propagation of sound waves. The continuity equation for a compressible fluid is

$$\frac{1}{\gamma_*} \frac{\partial p}{\partial t} = -\boldsymbol{\nabla} \cdot \mathbf{u},\tag{1}$$

where $1/\gamma_*$ is the compressibility and $\gamma_* = \rho c_s^2$. Here ρ is the density, p is the pressure, and **u** is the velocity.

If we vertically integrate the time discretized version of the continuity equation [see Eq. (A9) in the appendix], we obtain by use of the equation for the surface elevation [(A3)]

$$\frac{\eta^{n+1} - \eta^{n-1}}{2\Delta t} = -\sum_{i=1}^{N} \frac{\gamma}{a\cos\theta} \left[\frac{\partial U_i^n}{\partial \phi} + \frac{\partial}{\partial \theta} (V_i^n \cos\theta) \right], \quad (2)$$

where *n* refers to the time level, the sum is over all *N* model layers, and a factor γ has been included in analogy with (1). Other symbols are defined in the appendix. A leapfrog discretization is shown for the purpose of illustration only, since (2) is not actually used in the model. Instead, the surface elevation is calculated by (A3), and in the pressure gradient term for layer *j*, (A2), the surface elevation is multiplied by a free parameter γ , namely,

$$\boldsymbol{\nabla} \boldsymbol{P}_{j} = g \left\{ \boldsymbol{\gamma} \rho_{j} \boldsymbol{\nabla} \boldsymbol{\eta} - \sum_{i=1}^{j-1} \left[(\rho_{j} - \rho_{i}) \boldsymbol{\nabla} \boldsymbol{H}_{i} \right] \right\}.$$
(3)

In the next section, it will be shown how this parameter changes the surface gravity wave phase speed by a factor of the square root of γ . We will refer to γ as the GWR parameter.

b. The GWR parameter

For the linear model problem the vertical modes are found by computing eigenvectors for the matrix

$$a_{ji} = \left[\gamma - \frac{\rho_j - \rho_{\min(j,i)}}{\rho_j}\right] H_{0j}, \tag{4}$$

where H_{0j} is the thickness at rest of layer *j* and ρ_j is its density (Jensen 1996). The eigenvalues of (4) are the equivalent depths, so the corresponding phase speeds are found by multiplication of gravity and taking the square root. Similarly, for the reduced gravity model, the phase speeds are computed from the eigenvalues of the matrix

$$a_{ji}' = \left[\frac{\rho_{\min(j,i)}}{\rho_j} - \frac{\rho_i}{\rho_N}\right] H_{0j},\tag{5}$$

where ρ_N is the density of the abyssal layer at rest (Jensen 1993).

In (4), the case of undistorted physics corresponds to $\gamma = 1$, and choosing $0 < \gamma < 1$ will slow external gravity waves down. In order to keep real eigenvalues, that is, free wave solutions, it is necessary to choose the GWR parameter, γ , sufficiently large. As it is decreased, the barotropic wave speed approaches the first baroclinic wave speed, and the associated eigenvectors become aligned; that is, they will become identical. Se-

lecting a GWR parameter too small will result in two modes with complex conjugate eigenvectors and complex conjugate eigenvalues, one mode being damped and one growing mode being unstable.

For a two-layer model it has been shown (Jensen 1996) that the a_{ji} given by (6) will have real eigenvalues, that is, have stable solutions, if

$$\gamma \ge 4 \frac{\Delta \rho}{\rho} \alpha (1 - \alpha), \tag{6}$$

where $\Delta \rho$ is the density difference between layers 2 and 1, and α is the ratio of the upper-layer thickness to the total depth. The right-hand side of (6) has a maximum of $\Delta \rho / \rho$ for $\alpha = 1/2$. For a system with many layers, choosing $\gamma > \Delta \rho_{\text{max}} / \rho$, where $\Delta \rho_{\text{max}}$ is the maximum density difference between all layers, has been found by experiments to be sufficient for stability.

Why is the external mode slowed down? The method is equivalent to decreasing the density difference between air and water. The modified air density is

$$\rho_a' = \rho_0 (1 - \gamma), \tag{7}$$

so if for instance $\gamma = 1/64$, and $\rho_0 = 1000$ kg m⁻³, we find $\rho'_a = 984$ kg m⁻³. A physical interpretation of the limitation for accuracy is then that we need to keep the density contrast to the modified air density significantly higher than internal density differences in the ocean problem we want to model, that is, as the practical limit $\gamma \gg \Delta \rho / \rho_0$ suggests.

As a result of behaving more like an internal wave along this modified air–sea system, the sea surface deviations become much larger than in reality (by a factor $1/\gamma$), while keeping the pressure gradients fairly accurate. While this might suggest that the method is seriously flawed with respect to sea level, and make comparison with satellite or tide gauge sea level measurements impossible, a realistic elevation can be restored by multiplying the model sea level output with a factor γ (Hearn and Hunter 1987; Jensen 1996).

The effect of the GWR method on gravity waves and planetary waves depends on the stratification in the ocean. For a two-layer model, an analytical expression can be obtained (Jensen 1996), but for more realistic models we need to rely on a numerical solution. For this reason, the numerical model and its basic state will be introduced next, before we further discuss the effect of the method.

3. Numerical model

The ocean model is based on the hydrodynamical multilayer model by Jensen (1991, 1993), modified to include bottom topography (Jensen 1996) and mixed layer physics. Since the wave dynamics and oceanic adjustment on seasonal to interannual timescales essentially are wind driven, we use a version of the model without active thermodynamics in order to save computer time.



FIG. 1. Vertical model structure with bottom topography confined to the deepest layer.

The model equations are in spherical coordinates, so they also apply for length scales larger than the oceanic barotropic deformation radius. Consider an ocean consisting of N layers of uniform density as shown in Fig. 1. The layers are labeled with increasing numbers downward. The assumption that all layers have a positive thickness is made. This implies that layers are not allowed to surface or merge, and that the bottom topography is always in the lowest layer. The model formulation is given in the appendix and further details are given in Jensen (1998).

Model initialization, boundary conditions, and forcing

The versions used here have five layers with bottom topography confined to the deep abyssal layer or, alternatively, four active layers with an infinitely deep layer below (4.5-layer model). The horizontal resolution is ¹/₃° in the zonal and in the meridional directions. The average initial thickness is 80, 120, 250, and 600 m for layers 1-4, respectively. For the finite-depth case, the thickness of layer 5 varies initially between 150 and 4950 m, with an average of 3000 m. The densities of layers 1-5 are 1023.6, 1025.4, 1026.5, 1027.2, and 1028.2, respectively, with a unit of kg m^{-3} . Bottom topography is from the ETOPO5 dataset (National Oceanic and Atmospheric Administration 1988). Due to the constraint that the bottom topography must be within the the deepest layer, the total ocean depth is confined to be between 1200 and 6000 m (Fig. 2); that is, depths less than 300 m are considered land, and depths between 300 and 1200 m are set to 1200 m. For simplicity, all boundaries were closed.

Wind stress is climatological monthly mean winds from the European Centre for Medium-Range Weather



FIG. 2. Bottom topography for the Indian Ocean model. Depth is given in 1000-m intervals. Minimum depth is 1200 m and maximum depth is 6000 m.

Forecasts reanalysis and has been interpolated to the model grid using cubic splines in space, while linear interpolation in time is done to each model time step.

4. Effect on wave propagation

a. Gravity waves

Figure 3 shows the phase speed of the barotropic and the first baroclinic mode as function of the factor $\Gamma = 1/\sqrt{\gamma}$, by which external gravity waves are slowed down. For the initial stratification and average depth used in our Indian Ocean model, we find that for $\gamma = 1/833.25$ or a potential computational speedup of $\Gamma = 28.9$, the barotropic and first baroclinic modes become identical with a phase propagation of 4.67 m s⁻¹. Decreasing γ further makes the numerical calculation unstable due to the growing mode. Although only of theoretical interest, we see that the damping or growth rate becomes maximum near $\gamma = 1/1600$, after which it is slowly reduced (Fig. 3).

In the shallower areas of the basin, the external phase speed cannot be reduced as much as in the deeper parts without causing instability. For instance, for a layer 5 depth of 1000 or 100 m rather than 3000 m, the instability sets in for $\gamma = 1/564$ and $\gamma = 1/361$, respectively. However, the GWR parameter must be significantly larger than these theoretical limits in order to produce acceptable results. Using the guideline $\gamma > \Delta \rho_{max}/\rho$ as discussed earlier, we find for the stratification in our model that we should choose $\gamma > 1/223$, or a speedup factor less than 15. In order to push the limits of the method, we will include $\gamma = 1/256$ as one of the standard test cases.

Table 1 shows the phase speeds of the gravity waves for the experiments with GWR factors in the range 25 PHASE SPEED 26 15 15 5 -5 -15 0 10 20 30 40 50 (1/ γ)^{1/2}

1/400–1 and for the reduced gravity layer model. The phase speeds of the first baroclinic modes are increased

by 15% for the reduced gravity model, and by 10%, 5.7%, and 1.2% for the finite-depth cases with GWR factors of 1/400, 1/256, and 1/64, respectively. For high-

FIG. 3. Phase speeds for the barotropic mode and the first baroclinic mode as a function of the potential speedup factor, $\Gamma = 1/\gamma^{\nu/2}$, for the Indian Ocean model. For speedup factors less than 28.9, the barotropic mode (medium dash) approaches the first baroclinic mode (short dash) as γ is increased, and the imaginary part (dash–dot) remains zero. For larger values of Γ , two complex conjugate modes exist. Propagation is determined by the real part (solid), while the imaginary components correspond to a growing mode (long dash) and damped (dotted) modes. The numerical model is only stable for real phase speeds, so the maximum possible speedup is by a factor of 28.9 with the given model stratification.

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TABLE 1. Gravity phase speed for external and internal modes $(m \ s^{-1})$.

GWR factor	External	First internal	Second internal	Third internal	Fourth internal
1 1/64 1/256 1/400 4.5 layers	199.162 24.618 11.839 9.138 none	3.112 3.151 3.290 3.420 3.578	1.606 1.603 1.596 1.591 1.617	0.973 0.973 0.973 0.974 0.972	0.718 0.718 0.718 0.718 0.718 0.717

er-order baroclinic modes, the maximum error is less than 1%.

b. Planetary waves

Jensen (1996) demonstrated that the phase error is less for baroclinic Rossby waves than for baroclinic gravity waves. The error of the energy propagation, that is, the group velocity, is also small except for wavenumbers near the inverse of the (unmodified) radius of deformation. Here the group velocity is close to zero, and any deviation will obviously result in relatively large errors.

For the barotropic Rossby waves, short waves are distorted less than long waves. Atmospheric disturbances have meridional scales of about 1000 km, and barotropic Rossby waves with that wavelength will propagate with 60%, 70%, or 80% of the correct speed for $\gamma = 1/100$, $\gamma = 1/64$, or $\gamma = 1/36$, respectively (see Fig. 6 in Jensen 1996).

For variable forcing, the Rossby wave response is of critical importance. The maximum frequency of Rossby waves, that is, those with zero group velocity, is given by

$$\omega_{\max} = \frac{\beta c}{2f}.$$
 (8)

Tobis (1996), pointed out that at 30° poleward of the equator, the minimum period is about 2.6 days for barotropic Rossby waves. A GWR factor of $\gamma = 1/100$ would increase this period by a factor 10, and clearly be a lower limit for a γ that is able to provide a correct barotropic Rossby wave response to monthly forcing. However, it should be emphasized that the baroclinic response, even with a GWR factor as low as 1/100, is not significantly altered (Jensen 1996).

5. Experiments

The model is spun up from rest and the circulation after 3 yr is investigated. While this is not sufficiently long to spin up the flow to a quasiperiodic state in the subtropics, it is quite adequate to estimate the accuracy of the GWR method and makes it feasible to run several cases.

The model has been shown to give a general circulation in very good agreement with observations. For the Arabian Sea, Jensen (1991) demonstrated that the model currents agreed very well with observed features (Schott 1983; Knox and Anderson 1985), such as the reversal of the Somali Current, its northward volume transport; the Great Whirl, and its northward migration during the southwest monsoon. Potemra et al. (1991) found that the model gave currents in the Bay of Bengal in agreement with the available observations. For the equatorial circulation, Jensen (1993) found remarkably good agreement with the model subsurface currents and the deep jets reported by Luyten and Roemmich (1982), and with the surface currents from Rao et al. (1989). Comparisons with observations in the South Equatorial Current, the North Equatorial Current, and Monsoon Current are also discussed in Jensen (1993).

Figure 4 shows the currents in the upper layer and Fig. 5 shows its depth, which in this case corresponds to a maximum mixed layer depth, since there is no heating or precipitation used in the forcing.

The control run was done without any reduction in gravity wave speed and required a time step of 45 s. Runs with GWR parameter, γ , values of 1/25, 1/64, 1/100, and 1/144 used time steps of 200, 320, 360, and 450 s, respectively, while runs with γ values of 1/256 and 1/400 used a time step of 600 s. The reduced gravity model was also run with a time step of 600 s, although a 1800-s time step is numerically stable. This was done in order to keep the time truncation error and filtering characteristics the same as for the $\gamma = 1/256$ and $\gamma = 1/400$ cases, so that any difference between the three solutions is caused only by the differences in the treatment of the barotropic mode, that is, filtering versus slowed-down phase speeds.

For the purpose of demonstrating the errors associated with the method, we choose to concentrate on the area, $20^{\circ}-10^{\circ}$ S, $43^{\circ}-70^{\circ}$ E, where errors are typical for the computation. In the eastern part of the basin, errors are most often smaller, while they tend to be larger in the western part of the basin, where intense boundary currents exist. However, when computing the root-mean-square (rms) errors, as, for instance, shown in Fig. 6, the entire basin was used.

a. Upper-layer results

The upper-layer flows are only weakly affected by the bottom topography. This was confirmed by running the control case ($\gamma = 1$) with a flat bottom and the same average depth as in the full topography case. As mentioned in the introduction, reduced gravity models reproduce the observed flow very well, and an important objective was to investigate if using a small GWR factor, combined with bottom topography, will improve the solution. The rms error was calculated for all cases, based on differences to the control run. Figure 6 displays the rms error as a function of reduction of the external gravity speed, for the reduced gravity model case, and for a case ($\gamma = 1$)





FIG. 4. (top) Upper-layer currents during boreal winter and (bottom) boreal summer. Length of vectors is proportional to the square root of the current speed. Unit is $(m s^{-1})^{1/2}$.

where the ocean depth has been replaced by its average value. Not surprisingly, the error generally increases with increasing depth. The exception is the error in the layer 4 thickness anomaly, which is smaller than errors in thickness anomaly for layers 2 and 3 for all $\gamma < 1$ cases and for the reduced gravity model. It is also seen that the error in layer thickness for the $\gamma = 1/256$ case is about the same as for the reduced gravity case. Most importantly, it is not larger.

For the zonal and meridional transports, the $\gamma = 1/256$ model results are significantly better than the reduced gravity layer model results. Is this because some of the effects of the variable topography on the upper-layer flows indeed are captured or is it because a finite average

depth is introduced? To answer this question, the $\gamma = 1$ and the $\gamma = 1/256$ cases were repeated with a flat bottom, but with the same average depth as in the previous cases. The rms errors for the $\gamma = 1$ case with flat bottom is shown in Fig. 6 over the label flat, while the flat bottom case with $\Gamma = 16$ is shown with open symbols.

Comparing the two $\gamma = 1/256$ cases, we see a significant increase in error when excluding bottom topography. The increase ranges from 5.5% in rms error in layer 1 thickness anomaly to 53% in layer 4 thickness anomaly. The error increase for the transport components in layers 3 and 4 is about 30%. Using the $\gamma = 1$ with a flat bottom as a reference it is found that the errors for reduced gravity case are larger than for the γ



FIG. 5. (top) Upper-layer thickness anomaly during boreal winter and (bottom) boreal summer. Contour interval is 10 m.

= 1/256 flat bottom case, typically of the order 30% larger. Finally, we see from Fig. 6 that the rms errors in the case of a flat bottom and $\gamma = 1$ in general have errors in layer 1–3 transport components of the same order as for the $\gamma = 1/100$ case with bottom topography. For layer 4, the errors are larger than for the $\gamma = 1/256$ case with bottom topography.

In conclusion we find that using a small GWR factor does include important effects of the bottom topography, but also introduces significant errors.

What kind of differences are found for the upperlayer flows? Figure 7 shows typical differences in currents between runs with various configurations for the area $10^{\circ}-20^{\circ}$ S and $43^{\circ}-70^{\circ}$ E. Variable depth influences the baroclinic Rossby wave propagation,

$$\frac{\omega}{k} = -\left[\beta + f_0 \mathcal{A}_5^2 / D\left(\frac{1}{a}\frac{\partial D}{\partial \theta} - \frac{1}{ka\cos\theta}\frac{\partial D}{\partial \phi}\right)\right] \times (k^2 + l^2 + f_0^2 / c'^2)^{-1},$$
(9)

given here for the case of weak topographic slopes (Charney and Flierl 1981). Here ω is the frequency; k and l the zonal and meridional wavenumber, respectively; f_0 is the Coriolis parameter; and β is its the variation with latitude. The influence of bottom topography is through the effect of the bottom slopes, ∇D , reduced by \mathcal{A}_5^2 , the square of the amplitude ratio of each baroclinic mode in the deepest layer to the barotropic mode, and through large-scale modification of the value of the internal gravity wave speed c'.



FIG. 6. (top) Rms error for layer thickness, (middle) zonal transport, (bottom) and meridional transport as function of reduction factor in external gravity wave speed ($\Gamma = 1/\sqrt{\gamma}$). The points above the label inf deep show the rms error for the reduced gravity model case, while the label flat is below the error for the flat bottom case without any reduction in gravity wave speed. Units are m for layer thickness and $m^2 s^{-1}$ for transport components.

Since the vertical structure functions do change with γ , the effect of the bottom slopes will be different. The first baroclinic mode is most affected by topography. With the given model stratification, we find that for γ = 1, the amplitude in layer 5 is 20% of the barotropic mode, so the influence of bottom slopes on the first baroclinic mode is only 4% of that on the barotropic mode. For $\gamma = 1/36$, 1/64, 1/100, and 1/256, respectively, the relative amplitudes are 19.3%, 18.5%, 17.4%, and 12.3%, reducing the impact on bottom slopes to the range 3.7%-1.5%. Due to the internal gravity wave speed effect, westward propagating Rossby waves experience higher than average propagation speeds east of the region shown in Fig. 7, since the ocean to the east is deeper than average (Fig. 2). Northwestward currents associated with baroclinic Rossby waves (Fig. 7a; 20°-17°S, 60°-70°E) will arrive later in case of the flat bottom (Fig. 7c). This is seen by the eastward displacement of this current system compared to the case with bottom topography, while the reduced gravity case (Fig. 7d) has the wave arrive too early. Largest differences of this type occur in the western side of the basin, in part due to accumulation of phase errors.

Errors are larger in the thickness anomaly (Fig. 6). For our representative region the thickness anomaly fields are shown in Fig. 8. For $\gamma = 1/256$, the thickness anomaly field is only marginally better than for the flat bottom case or the reduced gravity model. This is to be expected, since the large artificial surface elevations as-



FIG. 7. Currents in layer 4 on 15 Jan after 1080 days of integration: (a) control ($\gamma = 1$) with bottom topography, (b) $\gamma = 1/256$ with bottom topography, (c) control ($\gamma = 1$) with flat bottom, and (d) reduced gravity model. The vectors are proportional to the square root of the current speed. Unit is (m s⁻¹)^{1/2}. Maximum vector shown corresponds to 0.3 (m s⁻¹)^{1/2} equivalent to a current of 0.09 m s⁻¹. Longer vectors are truncated.

sociated with the GWR method must affect the layer thickness.

The figures above emphasize the similarities rather than the differences between the solutions. To reveal the errors, the 3-month average (Nov–Jan) volume transports in layer 3 from the control run are shown in Fig. 9. Differences in transports between the control and solutions with $\gamma = 1/64$, with $\gamma = 1/256$, and with reduced gravity are shown in Figs. 9b–9d. The GWR method has clearly smaller errors than the reduced gravity solution, which have errors of order one. As noted above, much of the error is due to phase differences, caused by the infinite abyssal depth in the reduced gravity model.

The similarities of the time-dependent solutions through the season are shown in Fig. 10. The zonal current component along 10°S for layer 3 is shown for the control case, the $\gamma = 1/64$ and $\gamma = 1/256$ cases,



FIG. 8. Thickness anomalies (m) in layer 4 on 15 Jan after 1080 days of integration: (a) control ($\gamma = 1$) with bottom topography, (b) $\gamma = 1/256$ with bottom topography, (c) control ($\gamma = 1$) with flat bottom, and (d) reduced gravity model.

and for the reduced gravity case. The most significant differences are that the reduced gravity model, and, to a less extent, the $\gamma = 1/64$ model have increased magnitudes of the currents in the vicinity of 70°–80°E. Also note that the maximum eastward current in that longitude band occurs at day 200 for the reduced gravity case, while the control has a maximum at day 230 with magnitude reduced by about 30%.

b. Deep-layer results

If the solution in the upper layer is modeled well for a small value of γ , is it also the case for the deep flow? If this is not the case, there would be little advantage in using the GWR method compared to the reduced gravity model, which of course does not provide a solution for the deep flow. Figure 11 shows the rms error



FIG. 9. Three-month averages (Nov–Jan) of volume transport per unit area in layer 3 has been computed. The solution from the control $(\gamma = 1)$ with bottom topography is in (a). Differences between the control and the other cases are shown as in (b), $\gamma = 1/64$ with bottom topography; (c), $\gamma = 1/256$ with bottom topography; and (d), reduced gravity model. The vector length is proportional to the square root of the transport. Unit is $(m \text{ s}^{-1})^{1/2}$. Maximum vector shown corresponds to 8 $(m \text{ s}^{-1})^{1/2}$, equivalent to a transport of 64 m² s⁻¹. Longer vectors are truncated.

for layer 5 thickness anomaly and transport components as function of the speedup factor. The rms error for the thickness anomaly was rescaled by dividing by a factor of 10.

A control computation in an ocean with the same average depth, but a flat bottom, has significant larger errors than solutions with $\gamma = 1/256$ in the transport components. However, the errors associated with the deepest layer thickness anomalies are large (Fig. 11). What is causing the large error in the deepest layer, when γ is decreased? It is clearly associated with the large surface elevation anomalies inherent in the GWR method as discussed above and by Jensen (1996). In fact, if errors in transport components are to be small, we see from (A1)–(A3) that gradients in layer thickness must increase to compensate for a decrease of γ . Since large



FIG. 10. Longitude–time plot of layer 3 zonal velocity along 10°S during the third year of integration: (a) currents from the control ($\gamma = 1$) with bottom topography, (b) $\gamma = 1/64$ with bottom topography, (c) $\gamma = 1/256$ with bottom topography, and (d) reduced gravity model. Contour interval is 0.8 cm s⁻¹ with dashed contours for negative values.

gradients are found in the deeper ocean, we find the largest absolute error here.

A typical example of the differences between solutions with different values of γ is shown in Fig. 12. Notice the currents in the solution with $\gamma = 1/100$ are nearly identical to those in the control. For the $\gamma = 1/256$ case, we can identify differences such as a too strong northeastward flow near 19°S, 53°E and a too weak clockwise eddy in the region northwest of Madagascar. On the other hand, the flat bottom case (Fig. 12d) is clearly unsatisfactory. Again, errors in the layer 5 thickness anomalies associated with the GWR method are significant (Fig. 13), compared to the flat bottom case. In fact, only the case with $\gamma = 1/25$ has smaller errors, which also was seen from Fig. 11.



FIG. 11. Rms error for layer 5 thickness *H* (circles), zonal transport *U* (diamonds), and meridional transport *V* (plus) as function of reduction factor in external gravity wave speed ($\Gamma = 1/\sqrt{\gamma}$). The label flat is below the error for the flat bottom case without any reduction in gravity wave speed (filled symbols). The rms error for the thickness *H* has been divided by a factor of 10 for scaling. Units are m for layer thickness and m² s⁻¹ for transport components.

A key question is whether using a small GWR parameter will give a meaningful deep flow. Figure 14 shows the zonal transport in layer 5 after 3 yrs of integration. Even in the $\gamma = 1/256$ case, the overall spatial structure is improved compared to the flat bottom case, so topographic effects are represented fairly well. From Fig. 11 we know that the rms error indeed is smaller.

Finally, it is of interest to examine the accuracy of the time-dependent response. An important phenomenon in the Indian Ocean is the strong semiannual equatorial deep zonal currents as observed by Luyten and Roemmich (1982). For the upper layers, Jensen (1991) found very good phase relations between reduced gravity model solutions and the observations. Solutions from the reduced gravity model or from models with $\gamma =$ 1/256 and larger have no significant differences from the control run. A flat bottom case reveals that bottom topography does not play an important role for the upper layers. The response in the deep layer is shown in Fig. 15 for the cases $\gamma = 1$, $\gamma = 1/64$, and $\gamma = 1/256$ as well as for the flat bottom case with $\gamma = 1$. There is a strong influence of the Chagos-Laccadive Plateau along 73°E and to a less extent along the Ninety East Ridge on the strength of the zonal currents near the topography. This is reflected in the $\gamma = 1/256$ solution. However, for that case the currents overflowing the ridges are too intense, by about 25% compared to the control case. The zonal current just offshore the African coast reverses annually, with eastward flow during the southwest monsoon. In the control run, this takes place from day 130 to day 260. In the $\gamma = 1/256$ case, its duration is decreased to last from day 160 to day 240, while the flat bottom case has the correct duration, but the east-



FIG. 12. Currents in layer 5 on 15 Jan after 1080 days of integration: (a) control ($\gamma = 1$), (b) $\gamma = 1/100$, and (c) $\gamma = 1/256$. Solutions in (a)–(c) include full bottom topography. (d) Control ($\gamma = 1$) with flat bottom. The vectors are proportional to the square root of the current speed. Unit is (m s⁻¹)^{1/2}. Maximum vector shown corresponds to 0.2 (m s⁻¹)^{1/2} equivalent to a current of 0.04 m s⁻¹. Longer vectors are truncated.

ward flow begins earlier, at day 110, and ends at day 240. Another discrepancy in the $\gamma = 1/256$ solution is the absence of westward flow near the coast from day 330 to day 30 of the following year. On the other hand, we find that the $\gamma = 1/64$ solution only has minor differences from the control.

6. Summary and discussion

The theory behind the GWR method was briefly discussed. In principle the method is equivalent to increasing the density of the air above the ocean and, thereby, reducing the effect of gravity. The method is stable for a speedup in excess of one order of magnitude compared to the unmodified control case. However, in order to keep errors within acceptable limits, the speedup should be limited to a factor of 8–16, depending on the application.



FIG. 13. Thickness anomalies (m) in layer 5 on 15 Jan after 1080 days of integration: (a) control ($\gamma = 1$), (b) $\gamma = 1/100$, and (c) $\gamma = 1/256$. Solutions in (a)–(c) include full bottom topography. (d) Control ($\gamma = 1$) with flat bottom.

It was demonstrated that bottom topography improved upon the intermediate layer solution compared to the reduced gravity layer model, even with a speedup factor of 16. However, for the deep flow, it was found that speedups in excess of a factor 8 produced errors that in some cases are too large, if the abyssal flow is of interest. This is similar to what is found with implicit methods, where time-dependent solutions with Courant numbers over 10 generally have unacceptable large errors. Tobis (1996) also found that results with a speedup factor of 8 gave qualitatively very good results for idealized model domains, in particular for the Tropics. For forcing that includes synoptic weather systems, the barotropic response is lowpass filtered due to the existence of a maximum frequency given by (6). This should be



FIG. 14. Zonal transport (m² s⁻¹) in layer 5 on 15 Jan after 1080 days of integration: (top left) control ($\gamma = 1$), (bottom left) $\gamma = 1/100$, and (top right) $\gamma = 1/256$. These solutions include full bottom topography. (bottom right) Control ($\gamma = 1$) with flat bottom.

kept in mind if the high-frequency response is of interest.

It should also be pointed out that the GWR method can be applied to the barotropic mode in an ocean model that uses mode splitting, for instance, MICOM. Since the computational speedup only applies to the barotropic mode, the gain in efficiency becomes relatively unimportant as the number of layers increases. If we assume, as in the introduction, that the workload of solving for the barotropic mode is 30 times that of a single baroclinic layer, using a speedup factor of 8 would result in speedups of the entire model of factors of about 4, 3, and 2 for 5-, 10-, and 20-layer models, respectively.

However, the main advantage of the GWR method is its simplicity, which makes it straightforward to include bottom topography in reduced gravity models, and also allows easy implementation on massively parallel computers due to the explicit time integration scheme. The method can be applied to any ocean basin, but is particularly well suited for tropical oceans where the barotropic mode is less important. In that case a GWR factor of about 1/64 will in most cases give satisfactory results. For model testing and debugging an even smaller GWR factor can be used to save substantial computational resources. It is also useful for numerical experimentation where a large number of scenarios are being explored for flows of interest, or if different forcings are used. During the search, computations can be done efficiently using a small value of the GWR factor, which, after selecting cases of particular interest, can be increased for better accuracy.

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APPENDIX

Model Formulation

a. Momentum equations in transport form

For each layer we define the vertically integrated volume transport components, U_j and V_j . The thickness of layer *j* is H_j , and the vertically averaged density is ρ_j . The transport equation for U_j becomes

$$\frac{\partial U_{j}}{\partial t} + \frac{1}{a\cos\theta} \frac{\partial}{\partial\phi} \left(\frac{U_{j}^{2}}{H_{j}} \right) + \frac{1}{a} \frac{\partial}{\partial\theta} \left(\frac{U_{j}V_{j}}{H_{j}} \right) - \frac{2U_{j}V_{j}\tan\theta}{aH_{j}} - fV_{j}$$
$$= \frac{-H_{j}}{\rho_{j}a\cos\theta} \frac{\partial P_{j}}{\partial\phi} + w_{ej}U_{j+1}/H_{j+1} - w_{e(j-1)}U_{j}/H_{j}$$
$$+ \mathcal{T}_{j}^{\phi}, \qquad (A1)$$

with a similar equation for V_j . Here g is the acceleration of gravity, $f = 2\Omega \sin\theta$ is the Coriolis parameter where Ω is the rate of rotation of the earth, and \mathcal{T}_i represents



FIG. 15. Zonal velocity in layer 5 during the third year of integration: (a) control ($\gamma = 1$), (b) $\gamma = 1/64$, and (c) $\gamma = 1/256$. (d) Control ($\gamma = 1$) with flat bottom. Contour levels are 0.8 cm s⁻¹ with dashed contours for negative values.

turbulent stresses in each layer for the meridional, ϕ , or zonal, θ , component. In (A1) the vertically integrated pressure gradient is given by

$$\boldsymbol{\nabla} P_{j} = g \bigg(\gamma \rho_{j} \boldsymbol{\nabla} \boldsymbol{\eta} - \sum_{i=1}^{j-1} \big[(\rho_{j} - \rho_{i}) \boldsymbol{\nabla} H_{i} \big] \bigg), \quad (A2)$$

where the factor γ , if less than 1, slows down the phase speed for the barotropic gravity waves. With *N* layers the surface displacement η is given by

$$\eta = \sum_{i=1}^{N} (H_i - H_{0i}),$$
(A3)

and H_{0j} is the thickness of layer j at rest.

The parameterization of turbulent momentum fluxes is for the zonal direction

$$\mathcal{T}_{j}^{\phi} = \mathcal{F}_{j}^{\phi} + \frac{\tau_{j}^{\phi,t}}{\rho_{i}} - \frac{\tau_{j}^{\phi,b}}{\rho_{i}} + \mathcal{D}_{j}^{\phi}, \qquad (A4)$$

with a similar expression for the meridional direction. In (A4), \mathcal{F} represents the eddy stress due to unresolved horizontal scales of motion, and τ is the tangential stress due to vertical friction. The superscripts denote the ϕ or θ component and top or bottom of the layer, while \mathcal{D}_{i}^{ϕ} is due to internal stress within each layer. The form of these stress terms is analogous to anisotropic molecular viscosity:

$$\mathcal{F}_{j}^{\phi} = A_{j}H_{j}\left\{\nabla^{2}\left(\frac{U_{j}}{H_{j}}\right) - \frac{1}{a^{2}\cos^{2}\theta} \\ \times \left[\frac{U_{j}}{H_{j}}(1 - 2\cos^{2}\theta) + 2\sin\theta\frac{\partial}{\partial\phi}\left(\frac{V_{j}}{H_{j}}\right)\right]\right\} \\ - A_{4,j}H_{j}\nabla^{4}\left(\frac{U_{j}}{H_{j}}\right) \\ - \frac{\Delta t}{2}\left[\left(\frac{U_{j}}{H_{j}}\right)^{2}\frac{1}{a^{2}\cos^{2}\theta}\frac{\partial^{2}U_{j}}{\partial\phi^{2}} + \left(\frac{V_{j}}{H_{j}}\right)^{2}\frac{1}{a^{2}}\frac{\partial^{2}U_{j}}{\partial\theta^{2}}\right], \quad (A5)$$

with a similar expression for meridional direction. Here A_i is a constant eddy viscosity. The form of the differential harmonic operator on the velocity is that given by Semtner (1986). The biharmonic friction coefficient $A_{4,i}$ is constant for each layer, and the nonlinear, velocity-dependent eddy viscosity is of the form given by Abbott et al. (1981). The latter term is part of the thirdorder correction to the central, second-order finite-difference approximation of the nonlinear terms in the momentum equation. However, since the net effect is to selectively diffuse high velocity shear, it is appropriate to include them explicitly as an additional eddy viscosity. A harmonic eddy viscosity of 1000 m² s⁻¹ and a biharmonic viscosity of -5×10^9 m⁴ s⁻¹ were used in the model runs. Along coastal boundaries the vorticity transfer is controlled using a no-slip boundary condition.

The vertical stress for the zonal direction at the top of each layer is

$$\tau_{j}^{\phi,t} = \tau_{w}^{\phi} \delta_{1j} + \rho_{j} A_{z} \left(\frac{U_{j-1}}{H_{j-1}} - \frac{U_{j}}{H_{j}} \right) \frac{2}{H_{j-1} + H_{j}} (1 - \delta_{1j}),$$
(A6)

with equivalent expressions for the meridional direction and for the bottom layer. In these expressions, τ_w denotes a wind stress, δ_{ij} is the Kronecker delta, and A_z is the vertical diffusivity.

The term D_j represents a parameterization of smallscale turbulent stress within each layer, shown here for the zonal direction:

$$\mathcal{D}_{j}^{\phi} = -\frac{c_{\tau}}{H_{j}} \left(\frac{U_{j}}{H_{j}}\right)^{3}.$$
 (A7)

The term given by (A7) dissipates strong flow in thin layers, which is a simple parameterization for transition to turbulence within a layer.

Finally, the zonal wind stress component is computed from

$$\tau_{w}^{\phi} = \rho_{a} c_{d} [(U_{a} - U_{1}/H_{1})^{2} + (V_{a} - V_{1}/H_{1})^{2}]^{1/2} \times (U_{a} - U_{1}/H_{1}),$$
(A8)

with an equivalent expression for the meridional direction. In (A8) ρ_a is the density of air, U_a and V_a are the zonal and meridional wind velocity components, and c_d is a constant drag coefficient. Note that the stress is based on the differential velocity between the ocean and the wind rather than the wind velocity alone.

b. Continuity equation

The continuity equation becomes

$$\frac{\partial H_j}{\partial t} + \frac{1}{a\cos\theta} \left[\frac{\partial U_j}{\partial\phi} + \frac{\partial}{\partial\theta} (V_j\cos\theta) \right] = w_{ej} - w_{e(j-1)}.$$
 (A9)

The vertical transport velocity across the bottom of each layer is w_e . These terms arise from entrainment by shear instability and entrainment or detrainment by buoyancy forcing and wind forcing, that is,

$$w_{ej} = \delta_{jl} w_k + w_{sj} + w_{dj} + w_{nj}, \qquad (A10)$$

where the first term is active only for the top layer. The term w_{sj} , due to interfacial stress, is positive for a layer in case the bulk Richardson number becomes less than a critical Richardson number, Ri_c (McCreary and Kundu 1988; 1989). The shear entrainment velocity is calculated as

$$w_{sj} = H_j \frac{(\mathrm{Ri}_c - \mathrm{Ri})^2}{\tau_e \mathrm{Ri}_c \mathrm{Ri}} \Theta(\mathrm{Ri}_c - \mathrm{Ri}), \qquad (A11)$$

where Θ is the unit step function, which has the value 1 for positive arguments and 0 otherwise. The Richardson number Ri is calculated using finite differences in the vertical.

Where prolonged convergence occurs, the layer thickness may become rather large after some time. In the real ocean, sinking water is subducted when the heat flux into the ocean is positive. In order to limit the layer thickness, a detrainment velocity is defined as

$$w_{dj} = -\Theta(B) \frac{(H_j - H_{\max,j})^2}{\tau_d H_{\max,j}} \Theta(H_j - H_{\max,j}),$$
 (A12)

where τ_d is a timescale over which the detrainment takes place and *B* is the buoyancy flux. The depth where detrainment becomes active, $H_{\max,j}$, is a constant chosen for each layer.

c. Mixed layer physics

The uppermost layer is subject to Kraus–Turner mixing. The net production of turbulent kinetic energy (TKE), q, is given by

$$q = 2m_0 \exp\left(-\frac{H_1 f}{\kappa u_*}\right) u_*^3 - \exp\left(-\frac{H_1 f}{\mu u_*}\right) BH_1, \quad (A13)$$

where u_* is the friction velocity. The formulation follows that of Oberhuber (1993), with $m_0 = 1.2$ and $\kappa = 0.4$.

For negative production of TKE (q < 0); that is, the mixed layer is warming up or become less salty due to net precipitation. In that case we set q = 0 in (A13) and solve for a new equilibrium depth, H_{MO} , the Monin–Obukov depth. We have detrainment toward H_{MO} by defining

$$h_m = \xi_{\rm MO} \max[\min(H_{\rm MO}, H_1), h_{\rm min}] + (1 - \xi_{\rm MO})H_1.$$
(A14)

In order to keep a finite upper-layer depth, a minimum depth, h_{\min} , is specified. The minimum function ensures that negative production of TKE cannot deepen the mixed layer. The factor ξ_{MO} , in the range 0–1, is introduced to approach H_{MO} over a finite time rather than instantaneously. A value of $\xi_{MO} = 0.1$ is used.

When we have a positive production of TKE, $(q \ge 0)$, either due to cooling, evaporation, or wind stirring, the mixed layer entrains toward

$$h_m = \min\{H_1 + 2\rho_1 q \Delta t / [g(\rho_2 - \rho_1)H_1], H_{\max}\},$$
 (A15)

where H_{max} is a penetration depth scale for the surfacegenerated TKE, the same as the allowed maximum depth before detrainment takes place. The resulting detrainment/entrainment velocity from buoyancy and wind forcing is

$$w_k = (h_m - H_1)/(2\Delta t).$$
 (A16)

d. Arbitrary Langrangian Eulerian coordinate control

Very thin layers or negative layer thickness presents a numerical problem. The arbitrary Langrangian Eulerian (ALE) method (e.g., Benson 1992) consists of two main steps. A Lagrangian time step that moves the interface without any mass crossing it, followed by a remapping step that repositions the interface and computes the associated mass flux across it. In this model it has been implemented as follows.

The term w_{nj} in (A10) becomes nonzero if a layer thickness is outside preset numerical limits. In that case, excess divergence or convergence is computed as an entrainment or detrainment rate across the base of the layer and the interface is positioned at the level determined by this numerical limitation. As long as the divergence violates the preset criteria, the layer thickness is held fixed. However, the layer is not necessarily fixed in the vertical, since other layers are able to change their thickness. The formulation for the movement of coordinates is as follows. The first step increases the entrainment into a thin layer to be as strong as into the layer above, that is,

$$w_{n1j} = \Theta[w_{p(j-1)}]\{\max[w_{p(j-1)}, w_{pj}] - w_{pj}\} \\ \times \Theta(H_{\min, j} - H_j).$$
(A17)

The second step limits the detrainment to half the layer thickness or the entrainment to half the layer thickness below, that is,

$$w_{n2j} = [1 - \Theta(w_{n1j})] \max[w_{n1j}, -H_j/(2dt)] + \Theta(w_{n1j}) \min[w_{n1j}, H_{j+1}/(2dt)].$$
(A18)

Finally, the divergence and convergence are limited as follows in the continuity equation. Step one is a predicted new layer thickness based on a pure Lagrangian step:

$$H_{j}^{*} = H_{j}^{m} - \Delta t^{*} [-\nabla \mathbf{U} + w_{pj} + w_{n2j} - w_{e(j-1)}],$$
(A19)

where m = n or n - 1 and $\Delta t^* = \Delta t$ or $2\Delta t$, depending on whether the scheme applied is Euler forward or leapfrog. The second step is a remapping, which assures that the thickness stays between D_{\min} and D_{\max} by applying

$$w_{nj} = \Theta(H_j^* - D_{\min})\Theta(D_{\max} - H_j^*)w_{n2j} + \Theta(D_{\min} - H_j^*)(D_{\min} - H_j^*)/\Delta t^* + \Theta(H_j^* - D_{\max})(D_{\max} - H_j^*)/\Delta t^*.$$
(A20)

The minimum layer thickness permitted in this simulation was 20 m for all layers, while the maximum allowed layer thickness was 400 m for layer 1, 800 m for layers 2 and 3, and 900 m for layer 4. In the finite-depth case, layer 5 was allowed to be 6000 m. Only the minimum layer thickness was encountered in the results from this study.

The ALE method is quite promising for ocean modeling. At Los Alamos National Laboratory, an effort is underway to implement it in the Parallel Ocean Program (J. Dukowicz and K. Bryan 1997, personal communication), and a similar hybrid coordinate is being considered for MICOM (R. Bleck and S. Dean 1997, personal communication).

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